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(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2018

FIRST YEAR [BATCH 2017-20]

STATISTICS (General)

Paper : II

Date : 26/05/2018

Time : 11 am – 1 pm

Full Marks : 50

[Use a separate Answer Book for each group]

Group - A

1. Answer **any two** questions : [2×5]

   a) Suppose the three variables x_1, x_2 and x_3 satisfy the relation $a_1x_1 + a_2x_2 + a_3x_3 = K$, then prove that $r_{12} = \frac{a_3^2S_3^2 - a_1^2S_1^2 - a_2^2S_2^2}{2a_1a_2S_1S_2}$, where symbols have their usual meanings. Also prove that all the partial correlation coefficients will be equal to –1, provided a_1, a_2, a_3 are of same sign. [4+1]

   b) Prove that the standard error of estimate in case of multiple regression satisfies: $S_{1\cdot23}^2 = (1 - r_{1\cdot23}^2)S_1^2$ where symbols have their usual meaning. [5]

   c) In a trivariate distribution, it is found that $r_{12} = 0\cdot41$, $r_{13} = 0\cdot71$ and $r_{23} = 0\cdot5$. Obtain the multiple correlation coefficient $r_{1\cdot23}$ and partial correlation coefficients $r_{12\cdot3}$ and $r_{13\cdot2}$. [2+1·5+1·5]

   d) Define Correlation index. Show that the value of correlation index increases with degree of the polynomial taken as regression equation. [2+3]

2. Answer **any one** question : [1×15]

   a) What is an attribute? What is contingency table? How can you get an idea about the association of attributes from the contingency table?
   State three measures of association of attributes with their ranges. Write your idea about binary response and logistic regression. [2+2+3+6+2]

   b) What do you mean by 'multiple correlation Coefficient'? Taking three variables express multiple correlation coefficient in terms of simple correlation coefficients.
   In a trivariate distribution it is found that $r_{12} = 0\cdot578$, $r_{13} = 0\cdot726$, $r_{12\cdot3} = -0\cdot221$. Find r_{23}, $r_{13\cdot2}$, $r_{1\cdot23}$. [3+6+6]

Group - B

3. Answer **any two** questions : [2×5]

   a) Let $X \backsim$ Poisson (λ). For what value of X, is the p.m.f of X maximized? [5]

   b) i) X & Y jointly follow bivariate Normal distribution. Show that X & Y are independent if the correlation coefficient between them is zero. [2]

      ii) Find the density function of $Y = e^Z$, where $z \backsim N(\mu, \sigma^2)$. [3]

   c) Let $X \backsim N(\mu, \sigma^2)$. Find the value of c in terms of σ such that $P(\mu - c \le X \le \mu + c) = 0\cdot95$.
   [Given, $\int_0^{1\cdot96} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt = 0\cdot475$] [5]

d) i) A fair coin is thrown once, if it lands heads up, it is thrown a second time. Find the probability distribution of the total number of heads. [3]

ii) If there is a war every 15 years on an average, find the probability that there will be no war in 25 years. [2]

Answer <u>any one</u> question between <u>Question no. 4 & 5</u> : [1×15]

4. a) For two jointly distributed random variable X & Y, prove $E(Y) = EE(Y|X)$. [4]

b) Find out the points of inflection of the normal distribution with the density function :

$$f(x) = \frac{1}{\sqrt{18\pi}} e^{-\frac{(x-7)^2}{18}}, -\infty < x < \infty.$$ [3]

c) In a normal distribution, 46% of the items are over 40 and 90% are under 75. Find the mean and standard deviation of the distribution.

(Given that $\int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt = 0\cdot 54$ or 0·90 according as x = 0·10 or 1·28) [8]

5. a) Use Normal approximation to the binomial distribution to determine the probability that number of heads lies between 6 & 8 in 16 flips of a balanced coin. [5]

b) An oil company conducts a geological study that indicates that an exploratory oil well should have a 20% chance of striking oil. What is the probability that the third strike comes on the seventh well drilled? [5]

c) Buses arrive a bus stop according to an exponential distribution with rate $\lambda = 4$ buses/hour. If you arrived at 8.00am at the bus stop, what is the expected time of the next bus? [5]

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TABLE I ORDINATES AND AREAS OF THE DISTRIBUTION OF STANDARD NORMAL VARIABLE*

τ	$\phi(\tau)$	$\Phi(\tau)$	τ	$\phi(\tau)$	$\Phi(\tau)$	τ	$\phi(\tau)$	$\Phi(\tau)$
·00	·3989423	·5000000						
·01	·3989223	·5039894	·51	·3502919	·6949743	1·01	·2395511	·8437524
·02	·3988625	·5079783	·52	·3484925	·6984682	1·02	·2371320	·8461358
·03	·3987628	·5119665	·53	·3466677	·7019440	1·03	·2347138	·8484950
·04	·3986233	·5159534	·54	·3448180	·7054015	1·04	·2322970	·8508300
·05	·3984439	·5199388	·55	·3429439	·7088403	1·05	·2298821	·8531409
·06	·3982248	·5239222	·56	·3410458	·7122603	1·06	·2274696	·8554277
·07	·3979661	·5279032	·57	·3391243	·7156612	1·07	·2250599	·8576903
·08	·3976677	·5318814	·58	·3371799	·7190427	1·08	·2226535	·8599289
·09	·3973298	·5358564	·59	·3352132	·7224047	1·09	·2202508	·8621434
·10	·3969525	·5398278	·60	·3332246	·7257469	1·10	·2178522	·8643339
·11	·3965360	·5437953	·61	·3312147	·7290691	1·11	·2154582	·8665005
·12	·3960802	·5477584	·62	·3291840	·7323711	1·12	·2130691	·8686431
·13	·3955854	·5517168	·63	·3271330	·7356527	1·13	·2106856	·8707619
·14	·3950517	·5556700	·64	·3250623	·7389137	1·14	·2083078	·8728568
·15	·3944793	·5596177	·65	·3229724	·7421539	1·15	·2059363	·8749281
·16	·3938684	·5635595	·66	·3208638	·7453731	1·16	·2035714	·8769756
·17	·3932190	·5674949	·67	·3187371	·7485711	1·17	·2012135	·8789995
·18	·3925315	·5714237	·68	·3165929	·7517478	1·18	·1988631	·8809999
·19	·3918060	·5753454	·69	·3144317	·7549029	1·19	·1965205	·8829768
·20	·3910427	·5792597	·70	·3122539	·7580363	1·20	·1941861	·8849303
·21	·3902419	·5831662	·71	·3100603	·7611479	1·21	·1918602	·8863606
·22	·3894038	·5870644	·72	·3078513	·7642375	1·22	·1895432	·8887676
·23	·3885286	·5909541	·73	·3056274	·7673049	1·23	·1872354	·8906514
·24	·3876166	·5948349	·74	·3033893	·7703500	1·24	·1849373	·8925123
·25	·3866681	·5987063	·75	·3011374	·7733726	1·25	·1826491	·8943502
·26	·3856834	·6025681	·76	·2988724	·7763727	1·26	·1803712	·8961653
·27	·3846627	·6064199	·77	·2965948	·7793501	1·27	·1781038	·8979577
·28	·3836063	·6102612	·78	·2943050	·7823046	1·28	·1758474	·8997274
·29	·3825146	·6140919	·79	·2920038	·7852361	1·29	·1736022	·9014747
·30	·3813878	·6179114	·80	·2896916	·7881446	1·30	·1713686	·9031995
·31	·3802264	·6217195	·81	·2873689	·7910299	1·31	·1691468	·9049021
·32	·3790305	·6255158	·82	·2850364	·7938919	1·32	·1669370	·9065825
·33	·3778007	·6293000	·83	·2826945	·7967306	1·33	·1647397	·9082409
·34	·3765372	·6330717	·84	·2803438	·7995458	1·34	·1625551	·9098773
·35	·3752403	·6368307	·85	·2779849	·8023375	1·35	·1603833	·9114920
·36	·3739106	·6405764	·86	·2756182	·8051055	1·36	·1582248	·9130850
·37	·3725483	·6443088	·87	·2732444	·8078498	1·37	·1560797	·9146565
·38	·3711539	·6480273	·88	·2708640	·8105703	1·38	·1539483	·9162067
·39	·3697277	·6517317	·89	·2684774	·8132671	1·39	·1518308	·9177356
·40	·3682701	·6554217	·90	·2660852	·8159399	1·40	·1497275	·9192433
·41	·3667817	·6590970	·91	·2636880	·8185887	1·41	·1476385	·9207302
·42	·3652627	·6627573	·92	·2612863	·8212136	1·42	·1455641	·9221962
·43	·3637136	·6664022	·93	·2588805	·8238145	1·43	·1435046	·9236415
·44	·3621349	·6700314	·94	·2564713	·8263912	1·44	·1414600	·9250663
·45	·3605270	·6736448	·95	·2540591	·8289439	1·45	·1394306	·9264707
·46	·3588903	·6772419	·96	·2516443	·8314724	1·46	·1374165	·9278550
·47	·3572253	·6808225	·97	·2492277	·8339768	1·47	·1354181	·9292191
·48	·3555325	·6843863	·98	·2468095	·8364569	1·48	·1334353	·9305634
·49	·3538124	·6879331	·99	·2443904	·8389129	1·49	·1314684	·9318879
·50	·3520653	·6914625	1·00	·2419707	·8413447	1·50	·1295176	·9331928

TABLE I (*Contd.*)

τ	$\phi(\tau)$	$\Phi(\tau)$	τ	$\phi(\tau)$	$\Phi(\tau)$	τ	$\phi(\tau)$	$\Phi(\tau)$
1·51	·1275830	·9344783	2·01	·0529192	·9777844	2·51	·0170947	·9939634
1·52	·1256646	·9357445	2·02	·0518636	·9783083	2·52	·0166701	·9941323
1·53	·1237628	·9369916	2·03	·0508239	·9788217	2·53	·0162545	·9942969
1·54	·1218775	·9382198	2·04	·0498001	·9793248	2·54	·0158476	·9944574
1·55	·1200090	·9394292	2·05	·0487920	·9798178	2·55	·0154493	·9946139
1·56	·1181573	·9406201	2·06	·0477996	·9803007	2·56	·0150596	·9947664
1·57	·1163225	·9417924	2·07	·0468226	·9807738	2·57	·0146782	·9949151
1·58	·1145048	·9429466	2·08	·0458611	·9812372	2·58	·0143051	·9950600
1·59	·1127042	·9440826	2·09	·0449148	·9816911	2·59	·0139401	·9952012
1·60	·1109208	·9452007	2·10	·0439836	·9821356	2·60	·0135830	·9953388
1·61	·1091548	·9463011	2·11	·0430674	·9825708	2·61	·0132337	·9954729
1·62	·1074061	·9473839	2·12	·0421661	·9829970	2·62	·0128921	·9956035
1·63	·1056748	·9484493	2·13	·0412795	·9834142	2·63	·0125581	·9957308
1·64	·1039611	·9494974	2·14	·0404076	·9838226	2·64	·0122315	·9958547
1·65	·1022649	·9505285	2·15	·0395500	·9842224	2·65	·0119122	·9959754
1·66	·1005864	·9515428	2·16	·0387069	·9846137	2·66	·0116001	·9960930
1·67	·0989255	·9525403	2·17	·0378779	·9849966	2·67	·0112951	·9962074
1·68	·0972823	·9535213	2·18	·0370629	·9853713	2·68	·0109969	·9963189
1·69	·0956568	·9544860	2·19	·0362619	·9857379	2·69	·0107056	·9964274
1·70	·0940491	·9554345	2·20	·0354746	·9860966	2·70	·0104209	·9965330
1·71	·0924591	·9563671	2·21	·0347009	·9864474	2·71	·0101428	·9966358
1·72	·0908870	·9572838	2·22	·0339408	·9867906	2·72	·0098712	·9967359
1·73	·0893326	·9581849	2·23	·0331939	·9871263	2·73	·0096058	·9968333
1·74	·0877961	·9590705	2·24	·0324603	·9874545	2·74	·0093466	·9969280
1·75	·0862773	·9599408	2·25	·0317397	·9877755	2·75	·0090936	·9970202
1·76	·0847764	·9607961	2·26	·0310319	·9880894	2·76	·0088465	·9971099
1·77	·0832932	·9616364	2·27	·0303370	·9883962	2·77	·0086052	·9971972
1·78	·0818278	·9624620	2·28	·0296546	·9886962	2·78	·0083697	·9972821
1·79	·0803801	·9632730	2·29	·0289847	·9889893	2·79	·0081398	·9973646
1·80	·0789502	·9640697	2·30	·0283270	·9892759	2·80	·0079155	·9974449
1·81	·0775379	·9648521	2·31	·0276816	·9895559	2·81	·0076965	·9975229
1·82	·0761433	·9656205	2·32	·0270481	·9898296	2·82	·0074829	·9975988
1·83	·0747663	·9663750	2·33	·0264265	·9900969	2·83	·0072744	·9976726
1·84	·0734068	·9671159	2·34	·0258166	·9903581	2·84	·0070711	·9977443
1·85	·0720649	·9678432	2·35	·0252182	·9906133	2·85	·0068728	·9978140
1·86	·0707404	·9685572	2·36	·0246313	·9908625	2·86	·0066793	·9978818
1·87	·0694333	·9692581	2·37	·0240556	·9911060	2·87	·0064907	·9979476
1·88	·0681436	·9699460	2·38	·0234910	·9913437	2·88	·0063067	·9980116
1·89	·0668711	·9706210	2·39	·0229374	·9915758	2·89	·0061274	·9980738
1·90	·0656158	·9712834	2·40	·0223945	·9918025	2·90	·0059525	·9981342
1·91	·0643777	·9719334	2·41	·0218624	·9920237	2·91	·0057821	·9981929
1·92	·0631566	·9725711	2·42	·0213407	·9922397	2·92	·0056160	·9982498
1·93	·0619524	·9731966	2·43	·0208294	·9924506	2·93	·0054541	·9983052
1·94	·0607652	·9738102	2·44	·0203284	·9926564	2·94	·0052963	·9983589
1·95	·0595947	·9744119	2·45	·0198374	·9928572	2·95	·0051426	·9984111
1·96	·0584409	·9750021	2·46	·0193563	·9930531	2·96	·0049929	·9984618
1·97	·0573038	·9755808	2·47	·0188850	·9932443	2·97	·0048470	·9985110
1·98	·0561831	·9761482	2·48	·0184233	·9934309	2·98	·0047050	·9985588
1·99	·0550789	·9767045	2·49	·0179711	·9936128	2·99	·0045666	·9986051
2·00	·0539910	·9772499	2·50	·0175283	·9937903	3·00	·0044318	·9986501

TABLE I (*Contd.*)

τ	$\phi(\tau)$	$\Phi(\tau)$	τ	$\phi(\tau)$	$\Phi(\tau)$	τ	$\phi(\tau)$	$\Phi(\tau)$
3·01	·0043007	·9986938	3·21	·0023089	·9993363	3·41	·0011910	·9996752
3·02	·0041729	·9987361	3·22	·0022358	·9993590	3·42	·0011510	·9996869
3·03	·0040486	·9987772	3·23	·0021649	·9993810	3·43	·0011122	·9996982
3·04	·0039276	·9988171	3·24	·0020960	·9994024	3·44	·0010747	·9997091
3·05	·0038098	·9988558	3·25	·0020290	·9994230	3·45	·0010383	·9997197
3·06	·0036951	·9988933	3·26	·0019641	·9994429	3·46	·0010030	·9997299
3·07	·0035836	·9989297	3·27	·0019010	·9994623	3·47	·0009689	·9997398
3·08	·0034751	·9989650	3·28	·0018397	·9994810	3·48	·0009358	·9997493
3·09	·0033695	·9989992	3·29	·0017803	·9994991	3·49	·0009037	·9997585
3·10	·0032668	·9990324	3·30	·0017226	·9995166	3·50	·0008727	·9997674
3·11	·0031669	·9990646	3·31	·0016666	·9995335	3·51	·0008426	·9997759
3·12	·0030698	·9990957	3·32	·0016122	·9995499	3·52	·0008135	·9997842
3·13	·0029754	·9991260	3·33	·0015595	·9995658	3·53	·0007853	·9997922
3·14	·0028835	·9991553	3·34	·0015084	·9995811	3·54	·0007581	·9997999
3·15	·0027943	·9991836	3·35	·0014587	·9995959	3·55	·0007317	·9998074
3·16	·0027075	·9992112	3·36	·0014106	·9996103	3·56	·0007001	·9998146
3·17	·0026231	·9992378	3·37	·0013639	·9996242	3·57	·0006814	·9998215
3·18	·0025412	·9992636	3·38	·0013187	·9996376	3·58	·0006575	·9998282
3·19	·0024615	·9992886	3·39	·0012748	·9996505	3·59	·0006343	·9998347
3·20	·0023841	·9993129	3·40	·0012322	·9996631	3·60	·0006119	·9998409

*Abridged from Table 1 of *Biometrika Tables for Statisticians*, vol. I, with the kind permission of the Biometrika Trustees.

TABLE II STANDARD NORMAL DISTRIBUTION

Values of τ_α

α	0·05	0·025	0·01	0·005
τ_α	1·645	1·960	2·326	2·576